Certified Symmetry and Dominance Breaking for Combinatorial Optimisation

Bart Bogaerts, **Stephan Gocht**, Ciaran McCreesh, Jakob Nordström

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  goal: assign which aircraft fly
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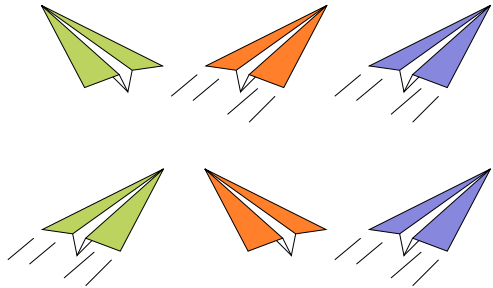

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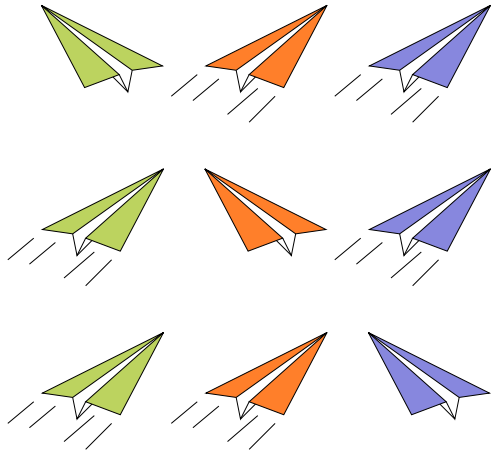

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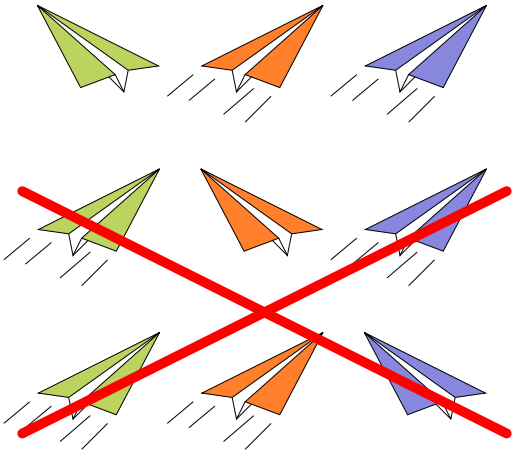

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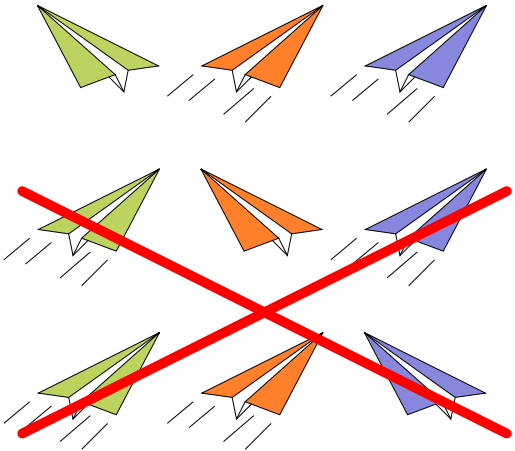
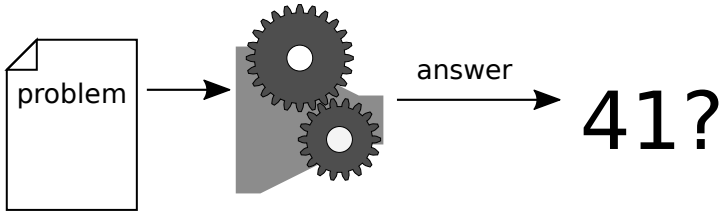


How can we know we didn't remove too many assignments?

Detecting Bugs with Certifying Algorithms



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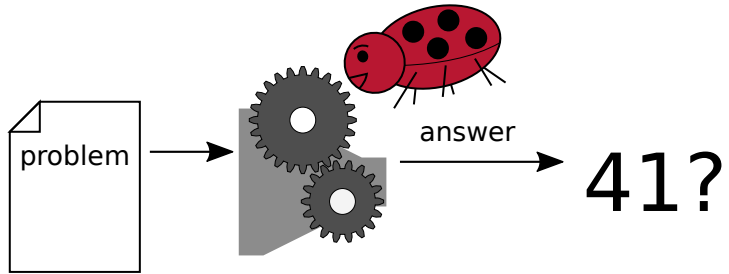

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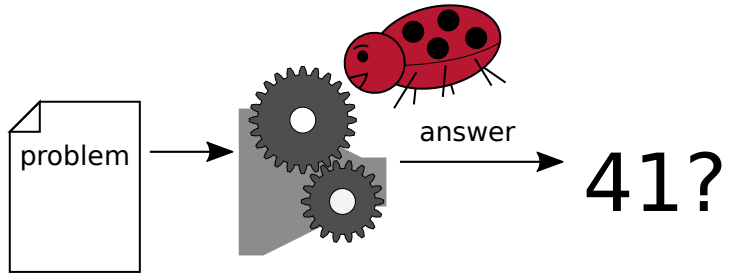

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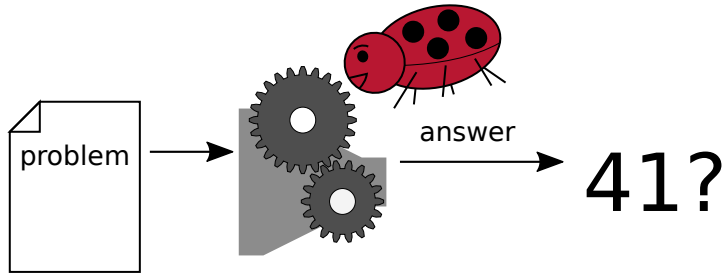

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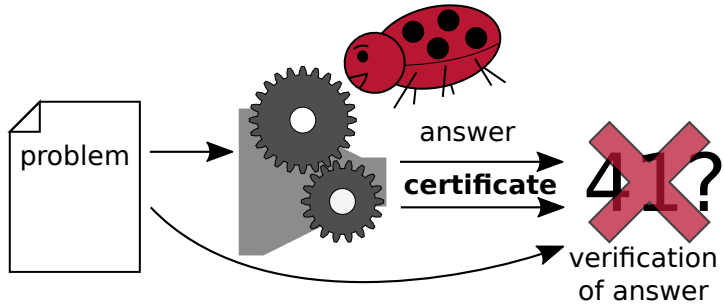

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  - simplifies testing: no ground truth needed
  - find bugs even if result is correct
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- while solving
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  - detect hardware errors
- after solving
  - analyse certificate to understand and improve solving process
  - could use certificate to audit solution afterwards

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But how?

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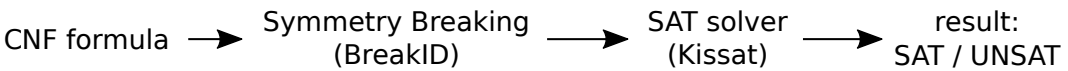

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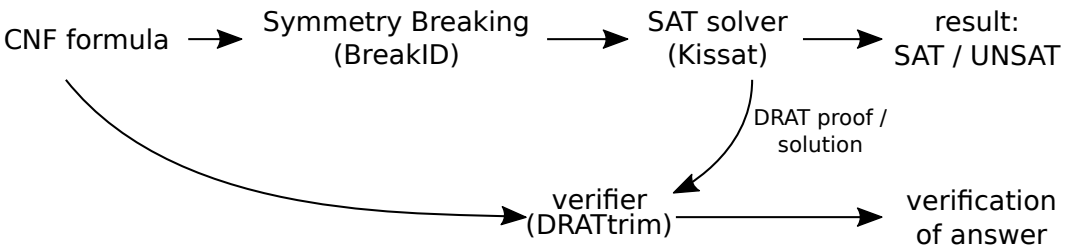

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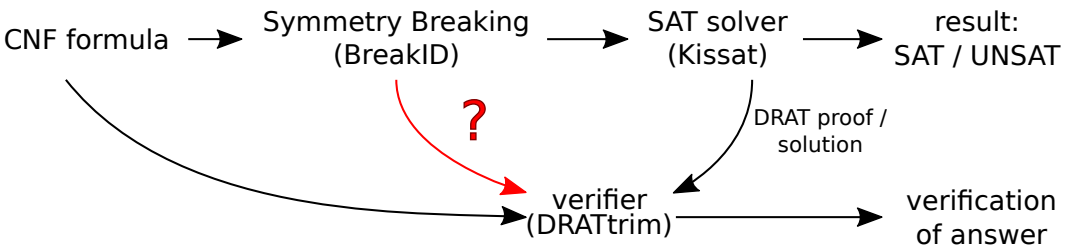

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  - approach based on DRAT proposed [HHW15]
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- How about practical proof logging for other solving paradigms?
  - MaxSAT solving
  - constraint programming (CP)
  - mixed integer programming (MIP)
  - algebraic reasoning / Gröbner basis computations
  - pseudo-Boolean satisfiability and optimization

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Need to look beyond DRAT!

New Proof Systems on the Rise

many new proof systems with implemented proof checkers:

- delete symmetry reverse unit propagation (DSRUP) [TD20]
- propagation redundancy (PR) [HKB17]
- practical polynomial calculus (PAC) [RBK18, KFB20]
- propagation redundancy for BDDs [BB21]
- Max-SAT resolution [PCH21]

Our Approach

- general-purpose proof format: pseudo-Boolean proofs (PBP)
- reference implementation of verifier: VeriPB[1]
- allows easy proof logging for
  - reasoning with 0-1 linear inequalities (by design)
  - all-different constraints [EGMN20]
  - subgraph isomorphism [GMN20]
  - clique and maximum common (connected) subgraph [GMM+20]
  - parity/ XOR reasoning [GN21]

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This Work

- proof logging for symmetry and dominance breaking
- applied to SAT, constraint programming and max clique solving
- full support for optimization

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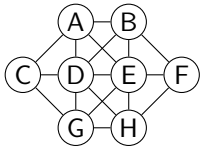
Supported Applications – Symmetry Breaking

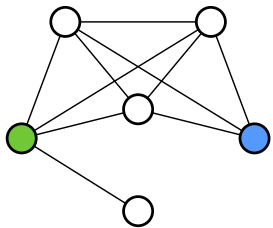
- SAT
  challenge: translate breaking constraint from PB to CNF
- Constraint Programming
  challenge: integer domains instead of 0-1



example: The Crystal Maze puzzle. Place numbers 1 to 8 without repetition, so that adjacent circles do not have consecutive numbers.

Supported Applications – Dominance Breaking

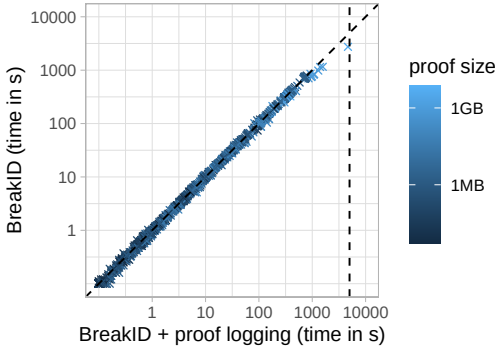
- maximum clique solving (find largest fully connected component)
  challenge: lazy breaking

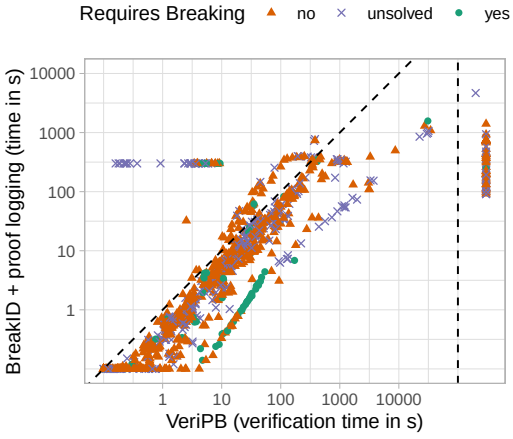


example: consider green but not blue node (every neighbour of blue is also neighbour of green)

Experiments

- evaluated on SAT competition benchmarks
- used BreakID[2] to find and break symmetries





[2] https://bitbucket.org/krr/breakid/

Running Example

$$\begin{aligned} \text{min: } & 4x_1 + 2x_2 + x_3 \\ \text{s.t. } & x_1 + x_2 + x_3 \geq 2 \end{aligned}$$

- boolean variable x is 0 (false) or 1 (true)
  (e.g. $x_1 = 1$ means green aircraft flies)
- pseudo-Boolean constraint:
  linear inequality over variables
- formula F: set of constraints
- objective function f to be minimized

Goal: find assignment minimizing objective and satisfying all constraints

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Claim:

- without loss of generality, add

$$3x_1 - 2x_2 - x_3 \leq 0$$

(to make solving easier)

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How can we know this claim is correct?

Naive Attempt: Proof by Truth Table

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$$3x_1 - 2x_2 - x_3 \leq 0 \quad (3)$$

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objective value	x_1	x_2	x_3
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Not practical, need something more efficient!

Output from Symmetry Breaking

- give formula $x_1 + x_2 + x_3 \geq 2$ to symmetry breaker

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- give formula $x_1 + x_2 + x_3 \geq 2$ to symmetry breaker
- output:
  - for permutation
    $\pi = \{ x_1 \mapsto x_2, x_2 \mapsto x_3, x_3 \mapsto x_1 \}$

objective value	x_1	x_2	x_3	
0	0	0	0	
1	0	0	1	
2	0	1	0	π
4	1	0	0	
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Notation: $C_{\restriction \pi}$ substitutes variables in C as specified by π

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    $(x_1 + x_2 + x_3 \geq 2)_{\restriction \pi} = x_2 + x_3 + x_1 \geq 2$
  - breaking constraint
    $3x_1 - 2x_2 - x_3 \leq 0$ (falsified by purple assignments)

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0	0	0	0	
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The Dominance Rule

- idea (from dominance breaking):
  - allow to add constraint C, e.g, $3x_1 - 2x_2 - x_3 \leq 0$
  - if for every ρ falsifying C but satisfying F (purple)
  - we find ρ' that satisfies F and $f(\rho) > f(\rho')$

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Dominance Rule (simplified)

Can derive constraint C from formula F if a *witnessing* substitution ω is provided such that

$$F \cup \{ \neg C \} \models F_{\restriction \omega} \cup \{ f > f_{\restriction \omega} \}$$

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5	1	0	1	↻
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Notation: $F \models F'$: satisfying assignment to F is also satisfying assignment to F'

The Dominance Rule — Example

- want to add $C\colon 3x_1 - 2x_2 - x_3 \leq 0$
- choose $\omega = \pi = \{ x_1 \mapsto x_2, x_2 \mapsto x_3, x_3 \mapsto x_1 \}$
- to prove: $F \cup \{ \neg C \} \models F_{\restriction\omega} \cup \{ f > f_{\restriction\omega} \}$, i.e.,

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  and $\neg C$: $3x_1 - 2x_2 - x_3 > 0$

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- given F: $x_1 + x_2 + x_3 \geq 2$
  and $\neg C$: $3x_1 - 2x_2 - x_3 > 0$
- derive $(x_1 + x_2 + x_3 \geq 2)_{\restriction\omega}$
  $\equiv \quad x_2 + x_3 + x_1 \geq 2$

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- to prove: $F \cup \{ \neg C \} \models F_{\restriction\omega} \cup \{ f > f_{\restriction\omega} \}$, i.e.,
- given F: $x_1 + x_2 + x_3 \geq 2$
  and $\neg C$: $3x_1 - 2x_2 - x_3 > 0$
- derive $(x_1 + x_2 + x_3 \geq 2)_{\restriction\omega}$
  $\equiv \quad x_2 + x_3 + x_1 \geq 2$

objective value	x_1	x_2	x_3	
0	0	0	0	
1	0	0	1	
2	0	1	0	
4	1	0	0	
3	0	1	1	
5	1	0	1	ω
6	1	1	0	ω
7	1	1	1	

The Dominance Rule — Example

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- derive
$$(x_1 + x_2 + x_3 \geq 2)_{\restriction\omega}$$
$$\equiv \quad x_2 + x_3 + x_1 \geq 2$$
- and
$$4x_1 + 2x_2 + x_3 > (4x_1 + 2x_2 + x_3)_{\restriction\omega}$$
$$\equiv 4x_1 + 2x_2 + x_3 > \quad 4x_2 + 2x_3 + x_1$$
$$\equiv 3x_1 - 2x_2 - x_3 > 0$$

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The Dominance Rule — Further Remarks

Dominance Rule (simplified)

Can derive constraint C from formula F if a *witnessing* substitution ω is provided such that

$$F \cup \{ \neg C \} \models F_{|\omega} \cup \{ f > f_{|\omega} \}$$

objective value	x_1	x_2	x_3	
0	0	0	0	
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- "$\models$" replaced by **efficiently machine-verifiable** proof system (cutting planes)

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- "$\models$" replaced by **efficiently machine-verifiable** proof system (cutting planes)
- in paper: any strict order instead of $f > f_{\restriction \omega}$

objective value	x_1	x_2	x_3	
0	0	0	0	
1	0	0	1	
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Conclusion

- proof logging is well-established standard for SAT solving
- so far, prohibitively expensive for some techniques (e.g. symmetry breaking)

3

4

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our work: proof logging for symmetry breaking (BreakID[3]) + verification (VeriPB[4])

- simple to implement + efficient proof checking
- fully evaluated for symmetry breaking on SAT competition benchmarks
- proof of concept for
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  - dynamic dominance breaking for maximum clique

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future work:

- proof logging for more techniques (e.g., in MaxSAT, CP or MIP)
- formally verified verifier

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